Problems in laser physics

Sheet 4

Handed out on 30. 11. 17 for the Tutorial on 7. 12. 17

Problem 10: Gaussian beams (5P)

A $P = 10$ W 532 nm TEM$_{00}$ beam of a frequency-doubled Nd^{3+}:YAG laser is focused to a spot radius of $w_0 = 50\ \mu$m. The beam intensity distribution follows from $I \propto |E|^2$ to be

$$I(r,z) = I_0 \frac{w_0^2}{w(z)^2} e^{-2\frac{r^2}{w(z)^2}} \ . \tag{1}$$

(a) Show that the peak intensity of the beam is given by

$$I_0 = \frac{2P}{\pi w_0^2} \ , \tag{2}$$

and calculate its actual value. (2P) Hint: You will need the "trick"

$$\int_a^b x e^{x^2} dx = \int_a^b \frac{1}{2}\frac{\partial}{\partial x} e^{x^2} dx = \frac{1}{2}\left[e^{x^2}\right]_a^b \ . \tag{3}$$

A thin lens with a focal length f will transform a Gaussian beam with a complex beam radius q_1 into a Gaussian beam with a complex beam radius q_2 according to

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \ . \tag{4}$$

(b) Show that this transformation only changes the radius of curvature of the beam's phase front and not its diameter measured directly after the lens (1P), i.e.

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f} \tag{5}$$

$$w_2 = w_1 \ . \tag{6}$$

(c) Show that a collimated Gaussian beam with a beam radius w_0 will be focused by a lens with focal length f to a spot radius w_0' at a distance d from the lens which are given by (2P)

$$
\begin{aligned}
d &= \frac{f}{1+\frac{\lambda^2 f^2}{\pi^2 w_0^4}} \qquad (7)\\
w_0' &= \frac{w_0}{\sqrt{1+\frac{\pi^2 w_0^4}{\lambda^2 f^2}}}\,. \qquad (8)
\end{aligned}
$$

Problem 11: Beam quality (3P)

The $1029\ \mathrm{nm}$ TEM_{00} beam of a Yb^{3+}:YAG laser is focused to a spot diameter of $2w_0 = 300\ \mu\mathrm{m}$. After the focus a divergence of $\theta = 5\ \mathrm{mrad}$ is measured.

(a) Determine the M^2 value of the beam. (1P)

(b) What is the equivalent wavelength λ' of a diffraction-limited beam that shows the same spot size versus divergence relation? (1P)

(b) What divergence θ' is necessary to focus the beam onto a $50\ \mu\mathrm{m}$ diameter spot? (1P)

Problem 12: Ray matrices of thin and thick lenses (4P)

Telescopes made from spherical lenses are often used to transform laser beams in size and divergence.

(a) Calculate the total ray matrix of a telescope consisting of two thin lenses of focal length f aligned concentrically and parallel to each other along the optical axis separated by a distance L and show that for $L = 2f$ a collimated ray is inverted with respect to the optical axis. (2P)
(b) Model a real lens as a block of glass with two spherical interfaces with a radius of curvature of R_1 and R_2, respectively, and a center thickness d. Calculate the total ray matrix for this thick lens. (1P)
(c) Let $d \to 0$ to derive "lens maker's formula", the relation between $\frac{1}{f}$ and the radii of curvature, by comparison with the thin lens ray matrix. (1P)